what different mean ionization in the scintillator as well as varying distributions (Landau distribution) for the different particle. Thus, in principle, one must attempt to accept all pulse amplitudes for the time spectra.

The competing requirements of high efficiency for all types of particles versus restricted pulse amplitude for good time resolution can be overcome with the use of multidimensional pulse-height analyzers. Thus, the time pulse might be stored as a function of pulse amplitude in each detector. Fast multidimensional analyzers were not available at the time of this experiment. However, by employing several single-channel analyzers and multiple coincidence circuitry (M.C.C.) we have been able to use the selective storage features of the RIDL pulse-height analyzer to effectively obtain a two by one-hundred-channel analyzer. As described in Sec. III1 of this paper, time pulses corresponding to a restricted range of pulse amplitudes in both counters could thus be stored in one section of the analyzer, whereas all pulse pairs not satisfying the limited range of pulse-amplitude selection resulted in storage of the time pulse in the second half of the analyzer memory. The improved time resolution in the restricted amplitude section is clearly seen in Fig. 2. Analysis of these spectra is performed in a straightforward manner by first obtaining the K^+/π^+ ratio in the solid curve, by inferring the number of unresolved K^+ in the opened circle curves, and then adding the total intensities of pions in both spectra and thereby ascertaining the absolute K^+ intensity. Without the device of memory split, the resolution of the K^+ peak from the π^+ is subject to much greater background corrections and at higher momenta is essentially impossible.

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Spin Rotation Coefficients in π-N Scattering

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The spin rotation coefficients of the recoil proton in π-N scattering are studied from the point of view of providing a means of resolving the present ambiguities in the phase-shift solution of the scattering cross section and the polarization of the recoil proton. There are six possible experiments four of which are independent, and the magnitude and the relative sign of the scattering matrix elements can uniquely be determined by four independent scattering experiments.

INTRODUCTION

THE advent of partially polarized proton targets[1] opens a new field of possibilities for nuclear experimentalists. Of particular interest to pion physicists is the possibility of measuring the spin rotation coefficients—A, R, A', and R' (Wolfenstein parameters)—of the recoil proton in π-N scattering.

It is well known that in the intermediate energy region (from 200 to 400 MeV) several ambiguities still persist in the phase-shift solution of π-p scattering even with the recently acquired polarization data.[2-4] Even though the present ambiguities can, in principle, be resolved by the polarization experiment,[5-8] due to various experimental difficulties[9] the polarization measurement at present is limited to a small angular region and the "resolving power" of the recoil proton polarization can not be fully utilized. In this paper we discuss several possible experiments which may be used in determining the phase shifts uniquely or in determining the scattering matrix elements.

DISCUSSIONS

A simple consideration based on the partial-wave analysis shows that a total of $2(2L+1)$ constants[10] are needed to describe each of the ten possible modes of

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[2] E. H. Rogers, O. Chamberlain, J. H. Foote, H. M. Steiner, C. Wiegand, and T. Ypsilantis, Rev. Mod. Phys. **33**, 356 (1961); J. H. Foote, O. Chamberlain, E. H. Rogers, H. M. Steiner, C. Wiegand, and T. Ypsilantis, Phys. Rev. **122**, 948, 959 (1961); E. L. Gregor'ev and N. A. Mitin, J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 413 (1960) [transaltion: Soviet Phys.—JETP **10**, 295 (1960)].

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[8] E. Fermi, Phys. Rev. **91**, 947 (1953).

[9] The large Coulomb scattering at small angles sets the lower limit and the analyzing power of the C^{12} analyzer which decreases rapidly below 100 MeV sets the upper limit on the angular region in which the polarization can be measured with any reasonable accuracy.

[10] Here L is the largest angular momentum state effecting the scattering. There are $2L+1$ real phase shifts and $2L+1$ amplitudes. For pion energies below 400 MeV the inelastic cross section is small and only the phase shifts need be determined.

π-N scattering. If the scattering is assumed to depend not on I_3, the third component of the composite isospin of the π-N system, only $4(2L+1)$ constants are needed to describe all modes of π-N scattering. Since a complete measurement of the differential cross section can yield $2L+1$ coefficients for each mode of scattering, a total of $10(2L+1)$ coefficients can, in principle, be obtained from the differential cross section measurements alone. Unfortunately, due to the particular way in which the phase shifts enter in the cross-section formula, several intrinsic ambiguities exist in the phase-shift solution of the differential cross section and one has to perform one, or possibly more, non-cross-sectional experiment.[8]

Since the composite πN system has four basis vectors, one would expect in principle 16 possible experiments for each of the 10 possible mode of π-N scattering.[11] However, if one subjects the scattering matrix M to the usual requirements of invariance under the Wigner time reversal, space rotation, and space inversion, only six of these can yield any useful information.[12] These can conveniently be defined in terms of five unit vectors $\mathbf{k}_1$ (incident pion direction), $\mathbf{k}_2$ (proton recoil direction), $\mathbf{n}(\mathbf{k}_1\times\mathbf{k}_2)$, $\mathbf{m}(\mathbf{n}\times\mathbf{k}_1)$, and $\mathbf{s}(\mathbf{n}\times\mathbf{k}_2)$. From parity considerations we can write[13]

$$I\langle\boldsymbol{\sigma}_f\rangle\cdot\mathbf{s}=I_0(A\mathbf{k}_1+R\mathbf{m})\cdot\langle\boldsymbol{\sigma}_i\rangle, \tag{1}$$

$$I\langle\boldsymbol{\sigma}_f\rangle\cdot\mathbf{k}_2=I_0(A'\mathbf{k}_1+R'\mathbf{m})\cdot\langle\boldsymbol{\sigma}_i\rangle, \tag{2}$$

$$I\langle\boldsymbol{\sigma}_f\rangle\cdot\mathbf{n}=I_0(P+1)\mathbf{n}\cdot\langle\boldsymbol{\sigma}_i\rangle, \tag{3}$$

where I is, of course, given by

$$I=I_0(1+\langle\boldsymbol{\sigma}_f\rangle\cdot\langle\boldsymbol{\sigma}_i\rangle). \tag{4}$$

From Eqs. (1) and (2) it is obvious that $A=-R'$ and $A'=R$. Now recalling that (see reference 12)

$$I\langle\boldsymbol{\sigma}_f\rangle=(|g|^2-|h|^2)\langle\boldsymbol{\sigma}_i\rangle+2\,\mathrm{Im}(gh^*)\mathbf{n}\times\langle\boldsymbol{\sigma}_i\rangle +2|h|^2\mathbf{n}\cdot\langle\boldsymbol{\sigma}_i\rangle\mathbf{n}+2\,\mathrm{Re}(g^*h)\mathbf{n}, \tag{5}$$

and comparing Eq. (5) with Eqs. (1) and (2) one can immediately write down the following relations:

$$I_0=|g|^2+|h|^2, \tag{6}$$

$$I_0P=2\,\mathrm{Re}(g^*h), \tag{7}$$

$$I_0R=(|g|^2-|h|^2)\cos\theta+2\,\mathrm{Im}(gh^*)\sin\theta, \tag{8}$$

$$I_0A=(|h|^2-|g|^2)\sin\theta+2\,\mathrm{Im}(gh^*)\cos\theta, \tag{9}$$

$$I_0R'=(|g|^2-|h|^2)\sin\theta+2\,\mathrm{Im}(gh^*)\cos\theta, \tag{10}$$

$$I_0A'=(|g|^2-|h|^2)\cos\theta+2\,\mathrm{Im}(gh^*)\sin\theta. \tag{11}$$

The spin rotation coefficients R, A, R', and A' relate the polarization of the proton target to the recoil proton polarization. Naturally, their measurements require polarized targets,[1] and the measurement of A' and R' will involve a spin-rotating magnet. The simplest way to measure A and R would be to observe the up-down asymmetry with the proton target polarized first along $\mathbf{k}_1$ for A coefficient and then along $\mathbf{m}$ for R coefficient. The up-down asymmetry then is related to these coefficients by

$$e_R=(I_{\mathrm{down}}-I_{\mathrm{up}})/(I_{\mathrm{down}}+I_{\mathrm{up}})=P_cR\langle\sigma_i\rangle_m, \tag{12}$$

$$e_A=(I_{\mathrm{down'}}-I_{\mathrm{up'}})/(I_{\mathrm{down'}}+I_{\mathrm{up'}})=P_cA\langle\sigma_i\rangle_{k_1}, \tag{13}$$

where P_c is the analyzing power of the analyzer. 10 or 20% polarization of the proton target should be sufficient for a reasonably accurate measurement of these coefficients.[2] According to Hwang *et al.*,[1] 10 or 20% polarized proton targets are not too far away.

Since only the polarization coefficient P contains terms odd in phase shifts, ambiguities which are mainly due to relative sign can only be resolved by the polarization experiment, while ambiguities which are mainly due to relative magnitude can best be resolved by the R and A experiments. R and A coefficients are more sensitive to the relative magnitude of phase shifts than the polarization. Most of the present ambiguities are due to relative signs as well as relative magnitudes and a suitable combination of P and the spin rotation coefficient experiments will be needed to resolve them.

Figure 1 shows the recoil proton polarization calculated using the phase shifts of Deahl *et al.*[7] at 224 MeV. The two Fermi sets whose major difference is in the sign of $I=\frac{1}{2}$ phase shifts predict practically the same polarization except for its sign. They predict very

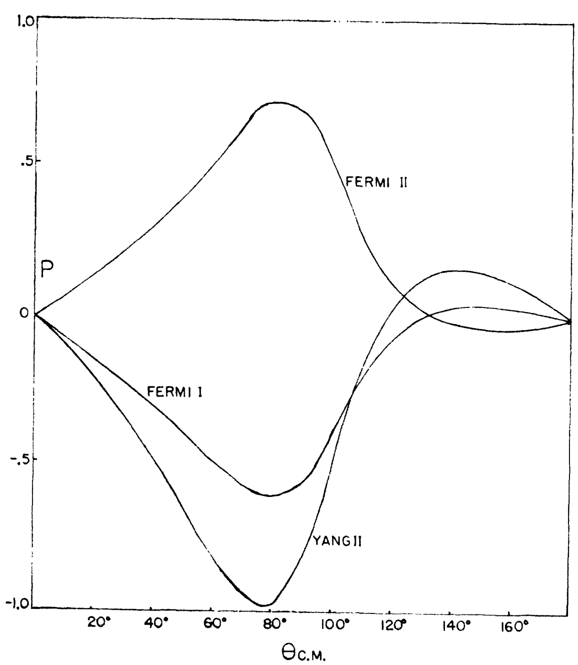

FIG. 1. **Recoil proton polarization at 224 MeV for π^--p scattering based on the phase shifts of Deahl *et al.* (reference 7).**

[11] H. M. MacGregor, M. J. Moravcsik, and H. P. Stapp, Ann. Rev. Nuclear Sci. **10**, 324 (1960); H. P. Stapp, University of California Radiation Laboratory Report, UCRL-3098, 1956 (unpublished).

[12] L. Wolfenstein, Phys. Rev. **96**, 1654 (1954).

[13] G. F. Chew, Rev. Mod. Phys. **33**, 361 (1961).

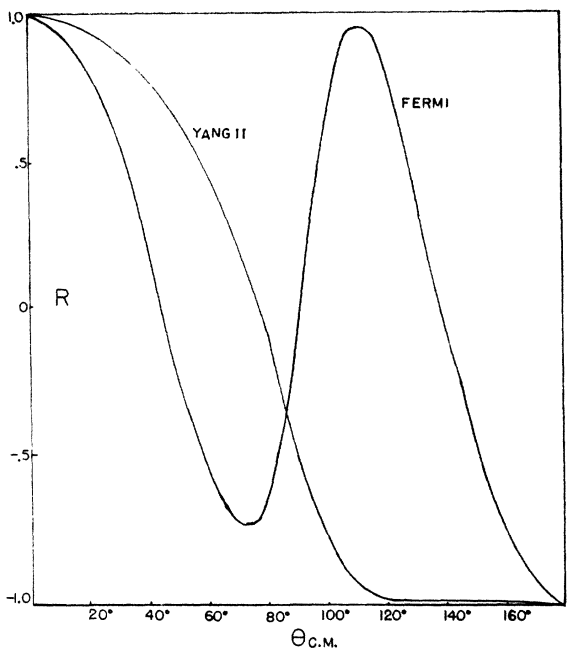

FIG. 2. R coefficient of the recoil proton in π^--p scattering at 224 MeV.

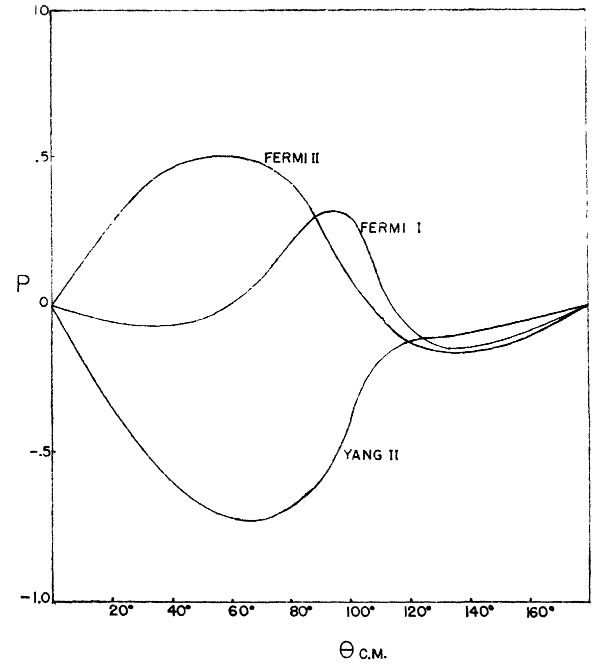

FIG. 4. Recoil proton polarization at 310 MeV for π^+-p scattering based on the phase shifts of Foote *et al.* (reference 2).

nearly identical values for A and R which are shown in Figs. 2 and 3 by a common curve. The Yang II set is similar to the Fermi I set except for the magnitude of P-state phase shifts. It is clear from these figures that the two Fermi sets cannot be resolved by A or R experiment. This is very unfortunate because in order to resolve this ambiguity one has to measure the polarization at small c.m. angles where C^{12} analyzers are impractical. One may obtain reasonable efficiency with spark chambers with sufficiently large solid angle using He analyzers at small angles. To date, no attempt of this kind has been made. The A and R experiments should resolve the Fermi-Yang II ambiguity more uniquely than the polarization experiment. They predict significantly different values for A and R at angles where the carbon analyzer is still usable.

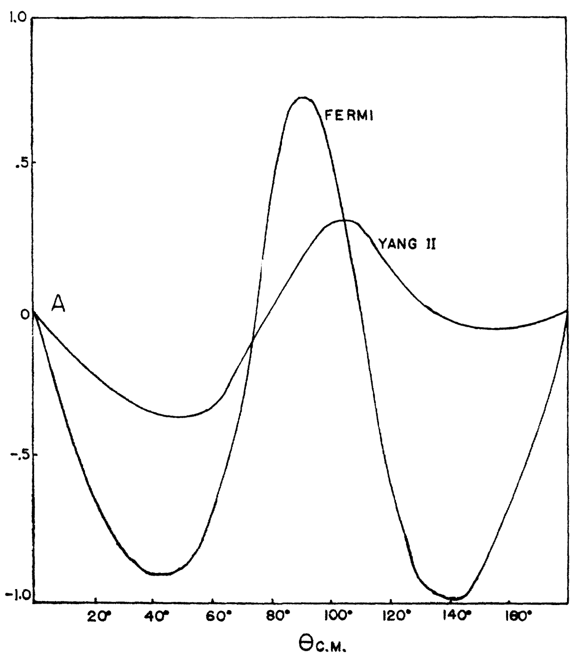

FIG. 3. A coefficient of the recoil proton in π^--p scattering at 224 MeV.

Figures 4, 5, and 6 show our calculation of P, A, and R coefficients using the phase shifts of Foote *et al.*[2] at 310 MeV. Their Fermi I and Fermi II sets differ mainly in D-wave phase shifts and take into account their polarization data near 130° c.m. From these figures it is evident that the A and R experiments have better resolving power than the polarization experiment.

At high energies it is expected that the partial-wave analysis becomes very inefficient and perhaps even invalid,[13,14] and a more direct approach such as the dispersion theoretic approach should be made. It is interesting to note that the combination of I_0, P, R, and A experiments determines the scattering matrix elements except for their phase. The magnitude of g and h and their relative phase are related to the experimentally determined values through the following

[14] M. J. Moravcsik, Phys. Rev. 118, 1615 (1960).

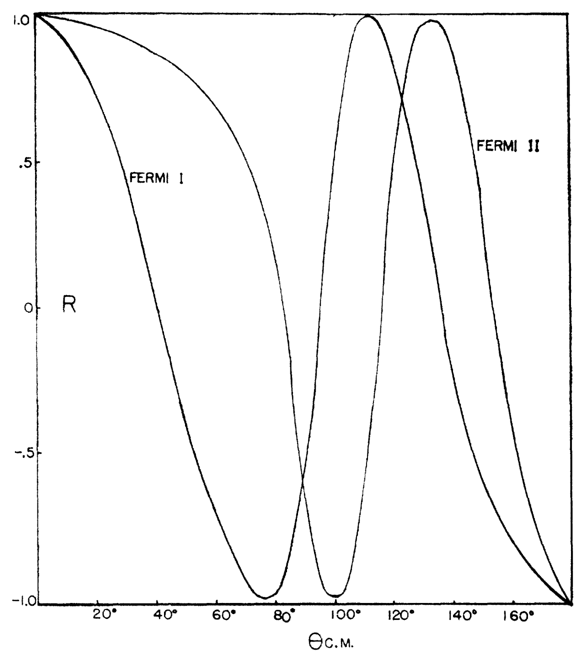

FIG. 5. R coefficient of the recoil proton in π^+-p scattering at 310 MeV.

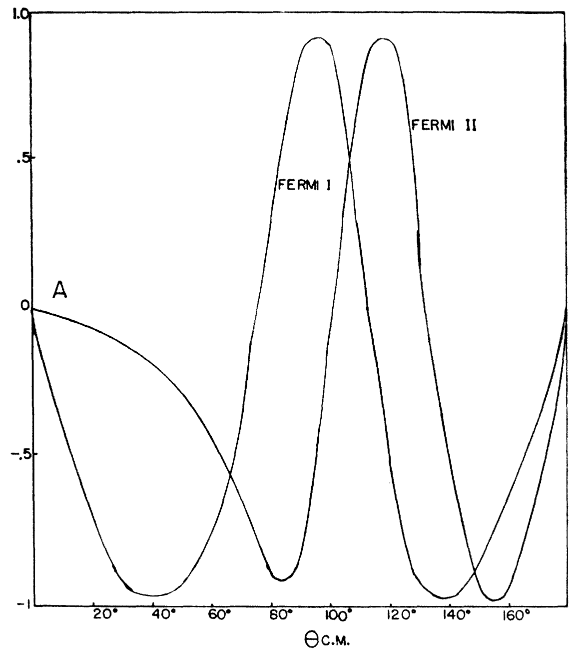

FIG. 6. A coefficient of the recoil proton in π^+-p scattering at 310 MeV.

relations:

$$|h|^2=\tfrac{1}{2}I_0(1-R\cos\theta-A\sin\theta), \tag{14}$$

$$|g|^2=\tfrac{1}{2}I_0(1+R\cos\theta+A\sin\theta), \tag{15}$$

$$gh^\dagger=\tfrac{1}{2}I_0(P+iR\sin\theta+iA\cos\theta). \tag{16}$$

The absolute phase has to be determined by other considerations.[11,2] Thus a complete measurement of I_0, P, A, and R at a given energy should determine uniquely the scattering matrix at that energy except for the absolute phase.

CONCLUSIONS

There are six possible experiments in π-p scattering. Four of these are independent and the combination of these four may be used in determining the scattering matrix elements. An explicit calculation shows that the R and A experiments cannot resolve the Fermi ambiguity at 224 MeV and one has to rely on the polarization experiment to resolve this ambiguity. The Yang ambiguity can best be resolved by the A and R experiments. At 310 MeV the Fermi ambiguity can be resolved by either measuring the polarization at small c.m. angles or by measuring A or R coefficient at large c.m. angles.

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